Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER				

FURTHER MATHEMATICS **9231/21**

Paper 2 Further Pure Mathematics 2 **May/June 2024**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined pages at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

DC (SL) 340584
© UCLES 2024

1 Find the roots of the equation $z^3 = -108\sqrt{3} + 108\mathrm{i}$, giving your answers in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $0 < \theta < 2\pi$. [5]

2 Find the Maclaurin's series for $e^{1+x^2} + e^{1-x}$ up to and including the term in x^2 . [4]

3 It is given that

$$x = \sin^{-1} t \quad \text{and} \quad y = t\cos^{-1} t, \quad \text{for } 0 \leqslant t < 1.$$

(a) Show that $\dfrac{dy}{dx} = -t + \sqrt{1-t^2}\cos^{-1} t$. [3]

(b) Find $\frac{d^2y}{dx^2}$ in terms of t. [4]

4 It is given that, for $n \geqslant 0$, $I_n = \int_0^{\ln 3} \operatorname{sech}^n x \, \mathrm{d}x$.

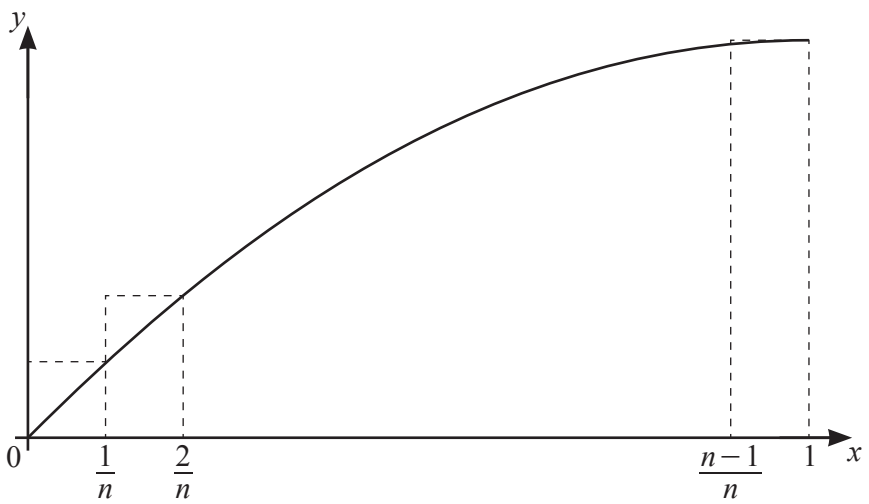
(a) Show that, for $n \geqslant 2$,

$$(n-1)I_n = \left(\frac{3}{5}\right)^{n-2}\left(\frac{4}{5}\right) + (n-2)I_{n-2}.$$ [5]

[You may use the result that $\frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{sech} x) = -\tanh x \operatorname{sech} x$.]

(b) Find the value of I_4. [3]

5



The diagram shows the curve with equation $y = 2x - x^2$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 (2x - x^2)\,dx < U_n$, where

$$U_n = \left(1 + \frac{1}{n}\right)\left(\frac{2}{3} - \frac{1}{6n}\right).$$ [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Use a similar method to find, in terms of n, a lower bound L_n for $\int_0^1 (2x - x^2)\,dx$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Show that $\lim_{n \to \infty} (U_n - L_n) = 0$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

6 **(a)** Show that $\left(\cosh x + \sinh x\right)^{\frac{1}{2}} = e^{\frac{1}{2}x}$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = 5\left(\cosh x + \sinh x\right)^{\frac{1}{2}},$$

given that, when $x = 0$, $y = 1$ and $\frac{dy}{dx} = \frac{4}{3}$. [10]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

7 **(a)** Use the substitution $u = 1 + x^2$ to find

$$\int \frac{x}{\sqrt{1+x^2}}\,\mathrm{d}x.$$ [2]

(b) Find the solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} - y = x^2 \sinh^{-1} x,$$

given that $y = 1$ when $x = 1$. Give your answer in the form $y = \mathrm{f}(x)$. [10]

8 **(a)** Find the set of values of a for which the system of equations

$$\begin{aligned} 6x + ay &= 3, \\ 2x - y &= 1, \\ x + 5y + 4z &= 2 \end{aligned}$$

has a unique solution. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Show that the system of equations in part **(a)** is consistent for all values of a. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 6 & 0 & 0 \\ 2 & -1 & 0 \\ 1 & 5 & 4 \end{pmatrix}.$$

(c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\left(14\mathbf{A} + 24\mathbf{I}\right)^2 = \mathbf{PDP}^{-1}$. [7]

(d) Use the characteristic equation of $\mathbf{A}$ to show that

$$\left(14\mathbf{A}+24\mathbf{I}\right)^2 = \mathbf{A}^4\left(\mathbf{A}+b\mathbf{I}\right)^2,$$

where b is an integer to be determined. [4]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

BLANK PAGE

BLANK PAGE

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

© UCLES 2024